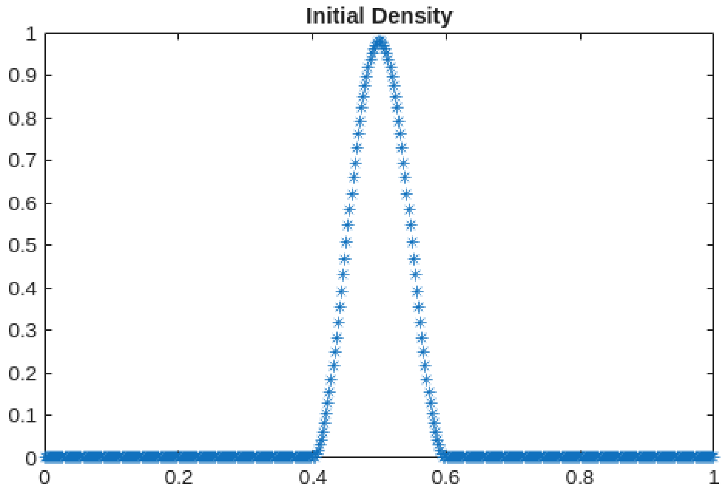
One Dimensional Eikonal using 5th WENO Fast Sweep Algorithm

After suffering from one more failure to make the 5th order WENO Algorithm for Two Eikonal Equation, i am moving on solve for the same problem with same algorithm, but with reduced dimension. The problem definition is given below:

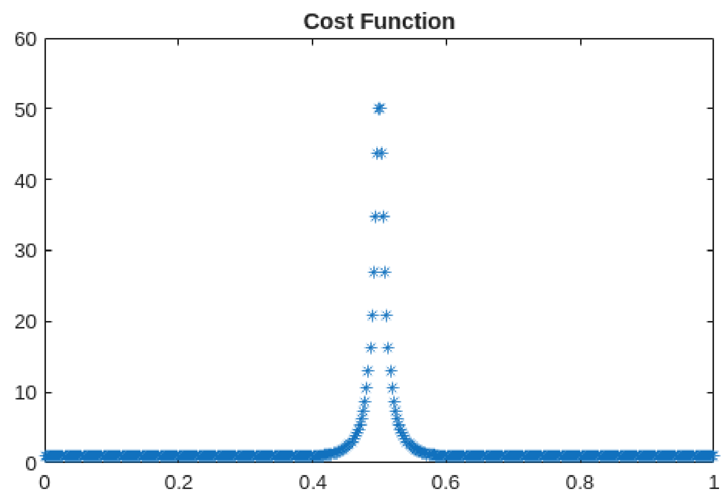
1.Problem Definition

So for simplicity, I will be taking the same problem as I have tajen in my previous problem.

```
%-----Parameters----%

N = 400;                % Grid Points
L = 1;                  % Lenght
dx = N/L;               % Grid Space
x = linspace(0,L,N);    % Domian

%-----Density Definition------%

rho_gauss = zeros(size(x));

for i = 1:1:N
     if x(i) >= 0.4 && x(i) <= 0.6
     % Raised cosine bump
     rho_gauss(i) = 0.49 * (1 + cos(pi * ((x(i) - 0.5) / 0.1)));
     end
     % Else, rho remains 0
end

%-----Speed and Discomfort Definition--%
u = zeros(size(x));
for i = 1:1:N
    u(i) = 1 - rho_gauss(i);
end

g = zeros(size(x));
for i = 1:1:N
    g(i) = rho_gauss(i)*rho_gauss(i);
end

%------Cost Function----%
c = zeros(size(x));
for i=1:1:N
    c(i) = 1/u(i) + g(i);
end
figure(1);
plot(x,rho_gauss,'*');
title('Initial Density')
```




```
figure(2);
plot(x,c,'*');
title('Cost Function');
```



2. 5th order WENO Fast Sweep Algorithm

Now is the time when i will be implementing the Fast Sweep algorithm. So given below is the eikonal euqation thta i will be solving as my governign equation. And this is just that **Magnitude of gradient of potential is equal to cost function**. So with that, being siad, I have also given below the algorithm in oine dimensional that i have derived from the paper.

The value of eta is taken to be:

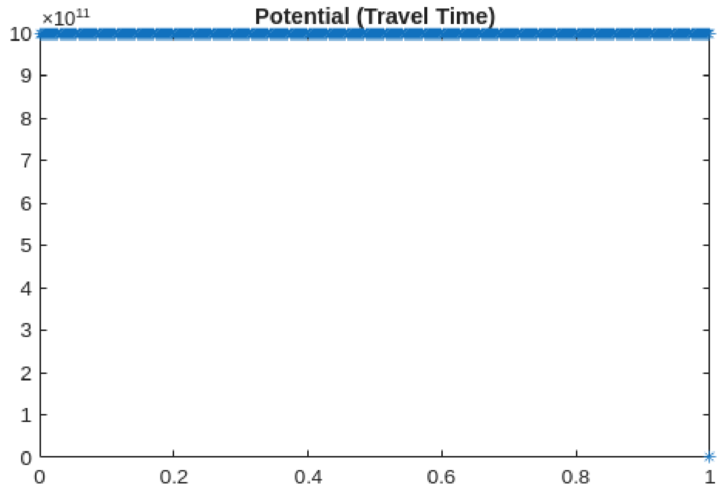
$$\beta_3 = \frac{13}{12}(s_i - 2s_{i+1} + s_{i+2})^2 + \frac{1}{4}(3s_i - 4s_{i+1} + s_{i+2})^2 .$$

re, ε is a parameter to prevent the denominator from becoming 0, and is usually taken as 10^{-6}. F

$_{i+1,j}$ is given by the following convex combination of the three third-order fluxes (31) as

The value of initila value of phi is taken to be inifinity and that is equal to normal numerical flux is needed there, and we simply set it to zero. The values of ρ at the ghost points inside the wall ar set to zero, and those of ϕ are set to 10^{12}.

```
phi = Fast_Sweep_Algorithm(c,N,L);
figure(3);
plot(x,phi,"*");
title('Potential (Travel Time)');
```




```
function phi = Fast_Sweep_Algorithm(cost,N,L)

    error_eta = 10^-9;
    eta = 10^-6;
    infinity = 10^12;
    h = L/N;

    %--Intialise the value of phi--%
    phi = infinity*ones(N);
    phi_old = zeros(N);

    % while sum(abs(phi_old - phi)) > error_eta
    for k = 1:1:10
        %--Updating phi values--%
        phi_old = phi;

        %--Directional Sweep--%
        for sweep = 1:2
            switch sweep
            case 1
                ix = 3:1:N-2;
            case 2
                ix = N-2:-1:3;
            end

            %--Boundary Condition--%
```

```
            phi(1) = phi(2);
            phi(N) = 0;

            %--Algorithm--%
            for i = ix
                r_back  = (eta + (phi(i) - 2*phi(i-1) + phi(i-2))^2) / (eta
+ (phi(i+1) - 2*phi(i) + phi(i-1))^2);
                r_front = (eta + (phi(i) - 2*phi(i+1) + phi(i+2))^2) / (eta
+ (phi(i+1) - 2*phi(i) + phi(i-1))^2);

                w_back  = 1 / (1 + 2*r_back^2);
                w_front = 1 / (1 + 2*r_front^2);

                phi_back    = (1 - w_back) * ((phi(i+1) - phi(i-1))/(2*h)) +
w_back * ((3*phi(i) - 4*phi(i-1) + phi(i-2))/(2*h));
                phi_front   = (1 - w_front) * ((phi(i+1) - phi(i-1))/(2*h))
+ w_front * ((-3*phi(i) + 4*phi(i+1) - phi(i+2))/(2*h));

                phi_min = min(phi(i) - h*phi_back , phi(i) + h*phi_front);

                if abs(phi_min) < cost(i)*h
                    phi(i) = phi_min + cost(i)*h;
                else
                    phi(i) = phi(i);
                end
            end
        end
    end
end
```