Two Dimensional WENO Advection Scheme

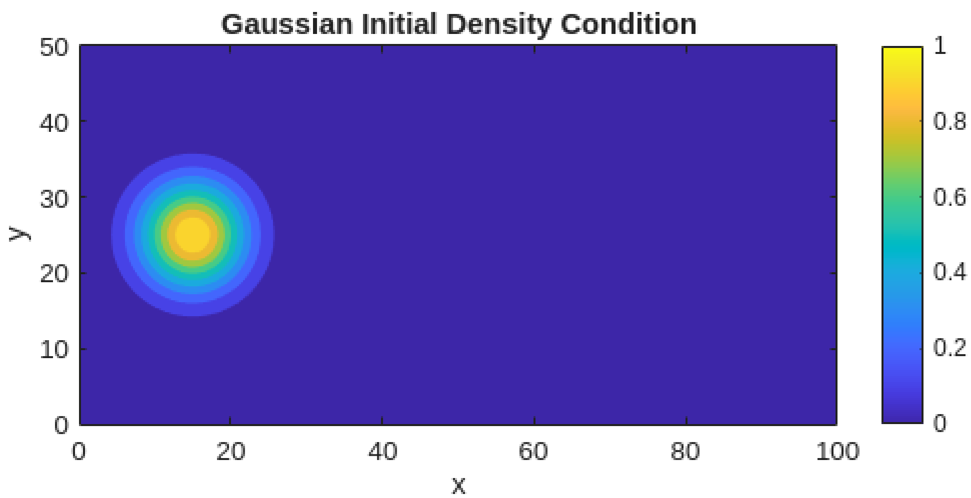
Now After successfully implementing the WENO fast sweep algorithm, i am now going to implement this 5th order WENo advection equation solver. So i will be takign two problems, and one of which is smmoth to test the workign of algorithm, and once thta is valided, i will be a takign a sharp disocontinutiy problme to test stable the algorithm is. Given below is the description of the first problem.

Problem Description

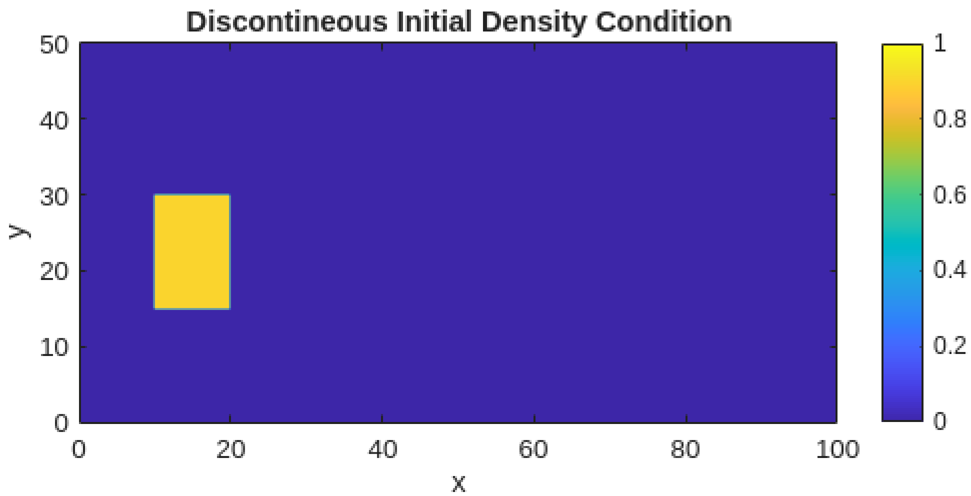
So the first problem is as follows. I am assumign a constant velocity in x dircetion and am interetsed in the numerical diffusion of the solution as it advects.

```
%-------Domain Parameter-----%
% Grid Points
global Nx;
global Ny;
Nx = 400;
Ny = 200;

global x_velocity y_velocity;
x_velocity = 1;
y_velocity = 0;

% Dimension of Domain
Lx = 100;
Ly = 50;

% Domain
x = linspace(0,Lx,Nx);
y = linspace(0,Ly,Ny);
[X,Y] = meshgrid(x,y);

% Space Interval
global h;
h = Lx/Nx;

% Total Time
t_total = 120;
% Time Interval
dt = 0.01;

%-------Initial Condition------%

% First Initial Condition (Gaussian Blob)

xc = 15;     % center x
yc = 25;     % center y
sigma = 5;   % width
rho_gaussian = exp( -((X-xc).^2 + (Y-yc).^2) / (2*sigma^2) );
```

```
% Second Intial Condition (Discontineous)
rho_square = zeros(size(X));
rho_square( X>=10 & X<=20 & Y>=15 & Y<=30 ) = 0.9;

% Plotting the Initial Condition
figure(1)
contourf(X,Y,rho_gaussian,'LineStyle', 'none');
title('Gaussian Initial Density Condition');
caxis([0,1]);
axis equal tight;
colorbar;
xlabel('x'); ylabel('y');
```



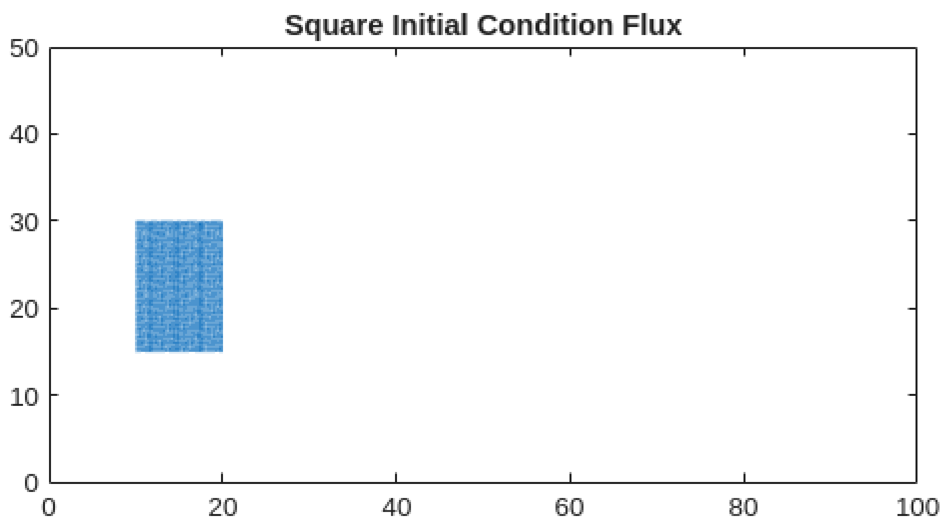

```
figure(2)
contourf(X,Y,rho_square,'LineStyle', 'none');
title('Discontineous Initial Density Condition');
caxis([0,1]);
axis equal tight;
colorbar;
xlabel('x'); ylabel('y');
```



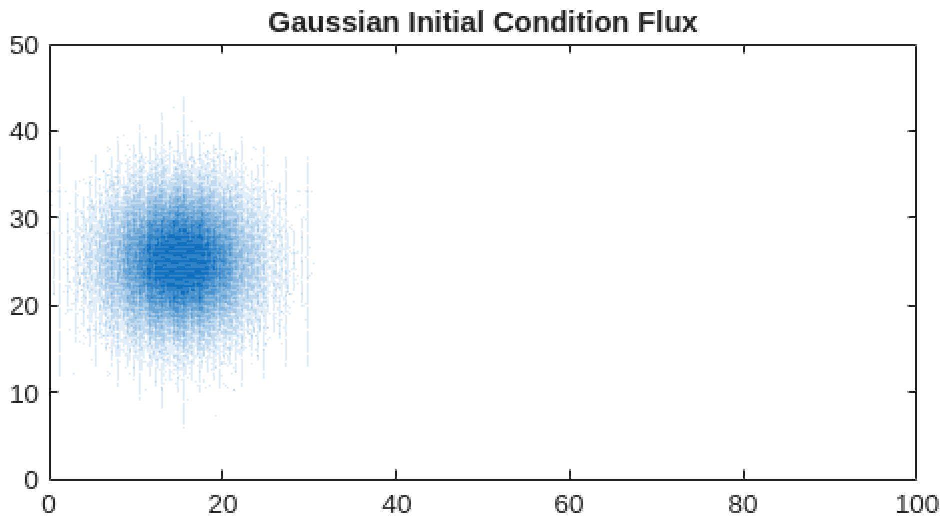
Now we will calculate the flux at each point for which i will have to assign the veloccity at each. point. So i say that everywhere the velocity of fluid is constant.A nd it is equal to **u = 1** and **v = 0**. Or the fluid is just flowing in x directuion everywhere, and there is no y component to the flow anywhere.

```
function flux = makeFlux(rho)
    global Nx Ny;
    global x_velocity
    global y_velocity;
    % Velocity Component
    x_velocity = 1;
    y_velocity = 0;

    % Initialzing Flux
    flux_x = zeros(Ny,Nx);
    flux_y = zeros(Ny,Nx);
    flux = zeros(2, Ny, Nx);

    for i = 1:1:Nx
        for j = 1:1:Ny
            flux_x(j,i) = rho(j,i) * x_velocity;
            flux_y(j,i) = rho(j,i) * y_velocity;

            flux(1,j,i) = flux_x(j,i);
            flux(2,j,i) = flux_y(j,i);
        end
    end
end

flux_square = makeFlux(rho_square);
flux_gaussian = makeFlux(rho_gaussian);
```

```
fx_square = squeeze(flux_square(1,:,:));
fy_square = squeeze(flux_square(2,:,:));

%---Plotting Flux---%

% Plotting Discontineous Flux

figure(4);
quiver(X, Y, fx_square, fy_square,0.5);
axis equal tight;
title('Square Initial Condition Flux');
```




```
% Plotting Gaussian Flux
fx_gauss = squeeze(flux_gaussian(1,:,:));
fy_gauss = squeeze(flux_gaussian(2,:,:));

figure(5);
quiver(X, Y, fx_gauss, fy_gauss,0.8);
axis equal tight;
title('Gaussian Initial Condition Flux');
```



It might not be clear,b ut these are arroaws, And they are scaled.

Now that the Flux is ready, and unstable, I will be making the function for WENO Adection equation.

One confusion i got was that if i will beb usign i+2 to i-2 to find the flux at i+1/2, then by replacing the i+1/2 with i-1/2, i will be using the values i-3 to i+1 to gte the values of flux at i-1/2. (Just replacing i with i-1). Now I am assuming that WENO is not symmetric, which means I must expect Upwind in WENO. I am still a bit skeptical about this, but lets see.

Time Marching

The loop given below is the time marching loop.

```
rho = rho_gaussian;

for t = 1:dt:t_total
    %--Making rho_n and RHS of rho_n
    rho_n = rho;
    semi_discritze_form_rho_n = makeRHS(rho_n);

    %--Makign the RHO_1 and then RHS of rho_1
    rho_1 = rho_n + dt * semi_discritze_form_rho_n;
    semi_discritze_form_rho_1 = makeRHS(rho_1);

    %-- Making the RHO_2 and RHS of RHO_2
    rho_2 = 3/4 * rho_n + 1/4 * (rho_1 + dt * semi_discritze_form_rho_1);
    semi_discritze_form_rho_2 = makeRHS(rho_2);

    %--Making the rho_next
    rho_next = 1/3 * rho_n + 2/3 * (rho_2 + dt * semi_discritze_form_rho_2);
```

```
    %--Ploting the rho_next;

    % Substituting back the rho value \
    rho = rho_next;
end
```

RHS of Semi discrite form

As in the paper, it uses the TVD Runga kutta time descritization. So here I will be desginign the gfunction for caluclating the L(rhgo). THis functiuon will take the value rho and then calcuate the RHS of the semi discrtise form of the adection equation fot hat rho.

```
function semi_discrite_form =  makeRHS(rho)

    global Nx Ny;
    semi_discrite_form = zeros(Ny-5,Nx-5);

    %--Flux for the given rho distribution
    flux = makeFlux(rho);
    flux_x = squeeze(flux(1,:,:));
    flux_y = squeeze(flux(2,:,:));

    % Calculating the maximum speed
    max_speed = 1; % (U = 1, V = 0)

    % -- Calculating the value of f+ and f- at {i+2,i+1,i,i-1,i-2,i-3} -- %
    fx_negative_whole = zeros(Ny,Nx);
    fx_positive_whole = zeros(Ny,Nx);
    fy_negative_whole = zeros(Ny,Nx);
    fy_positive_whole = zeros(Ny,Nx);

    fx_negative_half = zeros(Ny-5,Nx-5);
    fx_positive_half = zeros(Ny-5,Nx-5);
    fy_negative_half = zeros(Ny-5,Nx-5);
    fy_positive_half = zeros(Ny-5,Nx-5);

    for j = 1:1:Ny
        for i=1:1:Nx
            fx_positive_whole(j,i) = 1/2 * (flux_x(j,i) +
max_speed*rho(j,i));
            fx_negative_whole(j,i) = 1/2 * (flux_x(j,i) -
max_speed*rho(j,i));
            fy_positive_whole(j,i) = 1/2 * (flux_y(j,i) +
max_speed*rho(j,i));
            fy_negative_whole(j,i) = 1/2 * (flux_y(j,i) -
max_speed*rho(j,i));
        end
    end
```

```
% -- Computing the RHS of Semi Discrite form.
for i = 4:1:Nx-2
    for j = 4:1:Ny-2

        %--X--%

        % fx{+}_[i+1/2,j]
        fx_plus_x_half_forward = stencil( ...
            fx_positive_whole(j,i-2), ...
            fx_positive_whole(j,i-1), ...
            fx_positive_whole(j,i), ...
            fx_positive_whole(j,i+1), ...
            fx_positive_whole(j,i+2));

        % fx{+}_[i-1/2,j]
        fx_plus_x_half_backward = stencil( ...
            fx_positive_whole(j,i-3), ...
            fx_positive_whole(j,i-2), ...
            fx_positive_whole(j,i-1), ...
            fx_positive_whole(j,i), ...
            fx_positive_whole(j,i+1));

        % fx{-}_[i+1/2,j]
        fx_minus_x_half_forward = stencil( ...
            fx_negative_whole(j,i+2), ...
            fx_negative_whole(j,i+1), ...
            fx_negative_whole(j,i), ...
            fx_negative_whole(j,i-1), ...
            fx_negative_whole(j,i-2));

        % fx{-}_[i-1/2,j]
        fx_minus_x_half_backward = stencil( ...
            fx_negative_whole(j,i+1), ...
            fx_negative_whole(j,i), ...
            fx_negative_whole(j,i-1), ...
            fx_negative_whole(j,i-2), ...
            fx_negative_whole(j,i-3));

        %--Y--%

        % fy{+}_[i,j+1/2]
        fy_plus_y_half_forward = stencil( ...
            fy_positive_whole(j-2,i), ...
            fy_positive_whole(j-1,i), ...
            fy_positive_whole(j,i), ...
            fy_positive_whole(j+1,i), ...
            fy_positive_whole(j+2,i));

        % fy{+}_[i,j-1/2]
```

```
            fy_plus_y_half_backward = stencil( ...
                fy_positive_whole(j-3,i), ...
                fy_positive_whole(j-2,i), ...
                fy_positive_whole(j-1,i), ...
                fy_positive_whole(j,i), ...
                fy_positive_whole(j+1,i));

            % fy{-}_[i,j+1/2]
            fy_minus_y_half_forward = stencil( ...
                fy_negative_whole(j+2,i), ...
                fy_negative_whole(j+1,i), ...
                fy_negative_whole(j,i), ...
                fy_negative_whole(j-1,i), ...
                fy_negative_whole(j-2,i));

            % fy{-}_[i,j-1/2]
            fy_minus_y_half_backward = stencil( ...
                fy_negative_whole(j+1,i), ...
                fy_negative_whole(j,i), ...
                fy_negative_whole(j-1,i), ...
                fy_negative_whole(j-2,i), ...
                fy_negative_whole(j-3,i));

            fx_positive_half = fx_plus_x_half_forward +
fx_minus_x_half_forward;
            fx_negative_half = fx_plus_x_half_backward +
fx_minus_x_half_backward;
            fy_positive_half = fy_plus_y_half_forward +
fy_minus_y_half_forward;
            fy_negative_half = fy_plus_y_half_backward +
fy_minus_y_half_backward;

            % Computing RHS of Semi Discritized form
            semi_discrite_form(j-3,i-3) = - 1/h * (fx_positive_half -
fx_negative_half) - 1/h * (fy_positive_half - fy_negative_half);
        end
    end
end
```

This is the function used for stencil. So this will be used in calculating the value of

```
% To compute f half. Order of input matters
function f = stencil(f1,f2,f3,f4,f5)

    eta = 10^-6;

    % Compute S values
    s_1 = ((1/3) * f1) + ((-7/6) * f2) + ((11/6) * f3);
    s_2 = ((-1/6) * f2) + ((5/6) * f3) + ((1/3) * f4);
    s_3 = ((1/3) * f3) + ((5/6) * f4) + ((-1/6) * f5);
```

```
    % Compute Beta
    beta_1 = 13/12 * (f1 - 2*f2 + f3)^2 + 1/4 * (f1 - 4*f2 + 3*f3)^2;
    beta_2 = 13/12 * (f2 - 2*f3 + f4)^2 + 1/4 * (f2 - f4)^2;
    beta_3 = 13/12 * (f3 - 2*f4 + f5)^2 + 1/4 * (3*f3 - 4*f4 + f5)^2;

    % Definign Gamma
    gamma_1 = 1/10;
    gamma_2 = 3/5;
    gamma_3 = 3/10;

    % Computing weights bar
    w_1_dash = ( gamma_1 ) / ((eta + beta_1)^2);
    w_2_dash = ( gamma_2 ) / ((eta + beta_2)^2);
    w_3_dash = ( gamma_3 ) / ((eta + beta_3)^2);

    % Computing Weights
    w_1 = ( w_1_dash ) / (w_1_dash + w_2_dash + w_3_dash);
    w_2 = ( w_2_dash ) / (w_1_dash + w_2_dash + w_3_dash);
    w_3 = ( w_3_dash ) / (w_1_dash + w_2_dash + w_3_dash);

    % Computing Flux
    f = w_1 * s_1 + w_2 * s_2 + w_3 * s_3;
end
```